Spin structure in nonforward partons

Steven D. Bass
Physik Department, Technische Universität München, D-85747 Garching, Germany
and ECT, Strada delle Tabarelle 286, I-38050 Villazzano, Trento, Italy*
(Received 16 May 2001; published 27 March 2002)

Renormalization induces anomalous contributions in light-cone correlation functions. We discuss the role of the axial anomaly and gluon topology in nonforward parton distributions noting that nonforward matrix elements of the gluonic Chern-Simons current K_μ are not gauge invariant even in perturbation theory. The axial anomaly means that one has to be careful how to interpret information from hard exclusive reactions about the orbital angular momentum carried by the proton's internal constituents.

DOI: 10.1103/PhysRevD.65.074025 PACS number(s): 13.60.Fz, 12.38.Aw

I. INTRODUCTION

In a very interesting paper [1] Ji proposed that deeply virtual Compton scattering (DVCS) could be used to extract information about the total quark angular momentum in the proton. Ji's result has inspired large theoretical activity on nonforward parton distributions (also known as "skewed" or "generalized" distributions) [2–4]—for reviews see [5,6]—and proposals for new dedicated experiments at Jefferson Laboratory, DESY and COMPASS@CERN. The standard assumption implicit in this theoretical literature is that one can simply apply the formula $J_z=(S_z+L_z)$ to extract information about quark orbital angular momentum in the proton and that canonical arguments are applicable in the construction of the (spin dependent) distributions. The axial anomaly [7,8] means that various theoretical subtleties enter the derivation and that the "L_z number" may even depend on the physical process (polarized deep inelastic scattering or ν-p elastic scattering) used to extract "S_z." Given the large experimental interest in the DVCS process to help understand the internal spin structure of the proton, it is important to clarify this issue. In this paper we trace the anomaly through the derivation and clarify which quantities are "gold plated" and which are theorist, scheme or process dependent. Going from the forward to the nonforward spin distributions is nontrivial in that the nonforward (cf. forward) matrix elements of the anomaly current K_μ are not gauge-invariant even in perturbation theory.

II. THE SPIN STRUCTURE OF THE PROTON

To motivate our discussion of spin effects in DVCS we first briefly review what is known about the quark and gluonic intrinsic spin structure of the proton from the interpretation of polarized deep inelastic scattering data. We start with the simple sum rule for the spin $+\frac{1}{2}$ proton:

$$\frac{1}{2}=\frac{1}{2}\Sigma+L_z+S_z^g. \tag{1}$$

Here, $\frac{1}{2}\Sigma$ and S_z^g are the quark and gluonic intrinsic spin contributions to the nucleon's spin and L_z is the orbital contribution. One would like to understand the spin decomposition, Eq. (1), both in terms of the fundamental QCD quarks and gluons and also in terms of the constituent quark quasiparticles of low-energy QCD. In relativistic constituent quark models Σ is given by the flavor-singlet axial charge $g_A^{(0)}$. The value of $g_A^{(0)}$ extracted from polarized deep inelastic scattering experiments is $g_A^{(0)}|_{\rm pDIS}=0.2$–$0.35$ [9], roughly half of the value predicted by the constituent quark models. In QCD the interpretation of the individual quantities in Eq. (1) is quite subtle because of the axial anomaly [10], issues of gauge invariance [11,12,1] and dynamical $U_A(1)$ symmetry breaking [13–15,10]. The terms S_z^q, L_z^q and $J_z^g(=S_z^g+L_z^g)$ in the spin sum rule (1) can be expressed in terms of the proton matrix elements of gauge-invariant local operators with S_z^q identified with $g_A^{(0)}$, and L_z^q is identified with the proton matrix element of $[\bar{q}(\vec{z}\times\vec{D})_3 q](0)$, where D_μ is the gauge covariant derivative [1]. However, these quantities do not have a simple partonic interpretation: $g_A^{(0)}$ contains explicit gluonic information through the axial anomaly—see Eq. (2) below—which also introduces scale dependence into the flavor-singlet axial charge, and L_z^q is sensitive to gluonic degrees of freedom through the gauge-covariant derivative—for a recent discussion see [16].

In QCD the axial anomaly induces various gluonic contributions to $g_A^{(0)}$. Working in light-cone gauge $A_+=0$ one finds [17–20,13]

$$g_A^{(0)}=\left(\sum_q \Delta q-3\frac{\alpha_s}{2\pi}\Delta g\right)_{\rm partons}+\mathcal{C}. \tag{2}$$

Here $\frac{1}{2}\Delta q$ and Δg are the amount of spin carried by quark and gluon partons in the polarized proton and $\mathcal{C}$ measures the gluon-topological contribution to $g_A^{(0)}$ [21]. In Eq. (2) $\Delta q_{\rm partons}$ is associated with the hard photon scattering on quark and antiquarks with low transverse momentum squared, k_T^2 of the order of typical gluon virtualities in the proton, and $\Delta g_{\rm partons}$ is associated with the hard photon scattering on quarks and antiquarks carrying $k_T^2 \sim Q^2$. Jet studies and semi-inclusive measurements of the nucleon's spin-dependent g_1 structure function may be used to determine Δg and Δq for each flavor [22]. The topology term $\mathcal{C}$ is associated with dynamical axial U(1) symmetry breaking and has support only at Bjorken $x=0$. It is missed by polarized deep inelastic scattering experiments but could, in principle, be measured in elastic νp scattering [21]. An example of how to obtain a finite value for $\mathcal{C}$ is provided by Crewther's theory of quark-instanton interactions [23]. There, any instanton induced suppression of $g_A^{(0)}|_{\rm pDIS}$ (the axial charge carried by partons with finite momentum fraction $x>0$) is compensated by a shift of axial charge to the zero mode so that the total axial charge $g_A^{(0)}$ including $\mathcal{C}$ is conserved.

The quark and gluonic parton decomposition of $g_A^{(0)}$ in Eq. (2) is factorization scheme dependent. Equation (2) describes the singlet axial charge in the k_T^2 cutoff parton model and in the AB and JET schemes [24,25]. In the modified numerical subtraction ($\overline{\rm MS}$) scheme the polarized gluon contribution is absorbed into Δq so that $\Delta q_{\overline{\rm MS}}=(\Delta q-(\alpha_s/2\pi)\Delta g)_{\rm partons}$ [26]—see also [27].

The QCD $g_A^{(0)}$ is measured by the proton forward matrix element of the flavor singlet axial-vector current:

$$2mS_\mu g_A^{(0)}=\langle P,S|J^{GI}_{\mu 5}|P,S\rangle_c \tag{3}$$

where $|P,S\rangle$ denotes a proton state with momentum P and spin S and

$$J^{GI}_{\mu 5}=[\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s]_{GI} \tag{4}$$

is the gauge-invariantly renormalized singlet axial-vector operator. In each of the "partons," AB and JET schemes Δq and Δg are measured by the forward matrix elements of the partially conserved axial-vector current and gluonic Chern-Simons current contributions to J^{GI}_{+5} between parton states in the light-cone gauge $A_+=0$—see Secs. IV and V below. In light-cone gauge the forward matrix elements of the plus components of these gauge dependent currents are invariant under residual gauge degrees of freedom. However, the nonforward matrix elements of these currents are not invariant, even in perturbative QCD. This means that some of the schemes commonly used to analyze polarized deep inelastic data are not gauge invariant when "skewed" or extrapolated away from the forward direction.

In summary, the axial anomaly brings in gauge invariance issues and the possibility of zero-momentum contributions to the individual spin contributions in Eq. (1). How does this physics enter the theory of deeply virtual Compton scattering?

III. NONFORWARD PARTON DISTRIBUTIONS

Nonforward parton distributions are defined as the Fourier transforms of light-cone correlation functions. Consider an incident proton with mass M and momentum P_μ which gets given momentum $\Delta_\mu=(P'-P)_\mu$ and emerges into the final state with momentum P'_μ. We use $\bar{P}_\mu=\frac{1}{2}(P'+P)_\mu$ to denote the average nucleon momentum. In this paper we follow the notation of Radyushkin [2]. We let x denote the fraction of light-cone momentum $k_+=xP_+$ carried by a parton in the incident proton and $\zeta=\Delta_+/P_+$ denote the amount of light-cone momentum transfered to the proton.

In perturbative QCD the spin averaged cross section for deeply virtual Compton scattering receives contributions from both spin independent and spin dependent nonforward parton distributions. The formula suggested by perturbative QCD for the deeply virtual Compton amplitude $M^{IJ}=\mathcal{A}_{\rm DVCS}$ is

$$\begin{aligned}M^{IJ}(\vec{q}_\perp,\vec{\Delta}_\perp,\zeta)=-e_{\rm q}{}^2\frac{1}{2\bar{P}^+}\int_{\zeta-1}^{+1}dx[F_q(x,\zeta,t,\mu^2)\\ \times C_q(x,\zeta,Q^2,\mu^2)+F_g(x,\zeta,t,\mu^2)\\ \times C_g(x,\zeta,Q^2,\mu^2)\\ +\tilde{F}_q(x,\zeta,t,\mu^2)\tilde{C}_q(x,\zeta,Q^2,\mu^2)\\ +\tilde{F}_g(x,\zeta,t,\mu^2)\tilde{C}_g(x,\zeta,Q^2,\mu^2)]\\ +\mathcal{O}\left(\frac{1}{Q}\right).\end{aligned} \tag{5}$$

Here (I,J) refer to the polarization vectors ($\uparrow$ or $\downarrow$) of the initial and final state photons—here both are taken as $\uparrow$; F_q and F_g are the spin-independent nonforward quark and gluonic parton distributions and $\tilde{F}_q$ and $\tilde{F}_g$ are the spin-dependent parton distributions. These distributions are defined in Eqs. (7)–(10) below; μ^2 denotes the renormalization scale. In Eq. (5),

$$\begin{aligned}C_q^{\uparrow\uparrow}=C_q^{\downarrow\downarrow}=\left(\frac{1}{x-i\epsilon}+\frac{1}{x-\zeta+i\epsilon}\right)+O(\alpha_s)\\ \tilde{C}_q^{\uparrow\uparrow}=-\tilde{C}_q^{\downarrow\downarrow}=\left(\frac{1}{x-i\epsilon}-\frac{1}{x-\zeta+i\epsilon}\right)+O(\alpha_s)\end{aligned} \tag{6}$$

are the nonforward quark coefficient functions with $C_q^{\uparrow\downarrow}=C_q^{\downarrow\uparrow}=\tilde{C}_q^{\uparrow\downarrow}=\tilde{C}_q^{\downarrow\uparrow}=0$. The gluonic coefficients start at order α_s, viz. $C_g,\tilde{C}_g\sim O(\alpha_s)$. In DVCS ζ plays the role of the Bjorken variable $\zeta=x_{\rm Bj}$. Radyushkin [28] has derived conditions on the nonforward distributions which must be satisfied in order that QCD factorization holds beyond perturbation theory. One of these conditions is that the nonforward parton distributions do not contain any singular behavior at $x=0$ or $x=\zeta$. We shall return to this point in Sec. V below.

The spin independent nonforward quark and gluon distributions are

$$\begin{aligned}F_q(x,\zeta,t)=\int\frac{dy_-}{8\pi}e^{ixP_+y_-/2}\langle P'|\bar{q}(0)\gamma_+\\ \times\mathcal{P}e^{i\int_{y_-}^0 dy_-A_+}q(y)|P\rangle_{y_+=y_\perp=0}\\ =\frac{1}{2\bar{P}_+}\bar{U}(P')\Bigg[H_q(x,\zeta,t)\gamma_+\\ +E_q(x,\zeta,t)\frac{i\sigma_{+\alpha}(-\Delta^\alpha)}{2M}\Bigg]U(P)\end{aligned} \tag{7}$$

and

$$F_g(x,\zeta,t)=\frac{1}{xP_+}\int\frac{dy_-}{8\pi}e^{ixP_+y_-/2}\langle P'|G_{+\alpha}(0)\times\mathcal{P}e^{i\int_{y_-}^0 dy_-A_+}G_+^\alpha(y)|P\rangle_{y_+=y_\perp=0}$$

$$=\frac{1}{2\bar{P}_+^2}\bar{U}(P')\left[H_g(x,\zeta,t)\gamma_+ + E_g(x,\zeta,t)\frac{i\sigma_{+\alpha}(-\Delta^\alpha)}{2M}\right]U(P) \qquad (8)$$

respectively. In the forward limit $\zeta=0$ one recovers the spin independent quark and gluon distributions measured in deep inelastic scattering: $(q\pm\bar{q})(x)=\frac{1}{2}(H_q(x,0,0)\mp H_q(-x,0,0))$ and $g(x)=\frac{1}{2}(H_g(x,0,0)-H_g(-x,0,0))$ respectively. The spin dependent distributions are

$$\tilde{F}_q(x,\zeta,t)=\int\frac{dy_-}{8\pi}e^{ixP_+y_-/2}\langle P'|\bar{q}(0)\gamma_+\gamma_5\times\mathcal{P}e^{i\int_{y_-}^0 dy_-A_+}q(y)|P\rangle_{y_+=y_\perp=0}$$

$$=\frac{1}{2\bar{P}_+}\bar{U}(P')\left[\tilde{H}_q(x,\zeta,t)\gamma_+\gamma_5 + \tilde{E}_q(x,\zeta,t)\frac{1}{2M}\gamma_5(-\Delta_+)\right]U(P) \qquad (9)$$

and

$$\tilde{F}_g(x,\zeta,t)=-\frac{i}{xP_+}\int\frac{dy_-}{8\pi}e^{ixP_+y_-/2}\langle P'|G_{+\alpha}(0)\times\mathcal{P}e^{i\int_{y_-}^0 dy_-A_+}\tilde{G}_+^\alpha(y_-)|P\rangle_{y_+=y_\perp=0}$$

$$=\frac{1}{2\bar{P}_+^2}\bar{U}(P')\left[\tilde{H}_g(x,\zeta,t)\gamma_+\gamma_5 + \tilde{E}_g(x,\zeta,t)\frac{1}{2M}\gamma_5(-\Delta_+)\right]U(P). \qquad (10)$$

In the forward limit $\zeta=0$ one recovers the spin dependent quark and gluon distributions measured in deep inelastic scattering: $\Delta(q\pm\bar{q})(x)=\frac{1}{2}(\tilde{H}_q(x,0,0)\pm\tilde{H}_q(-x,0,0))$ and $\Delta g(x)=\frac{1}{2}(\tilde{H}_g(x,0,0)+\tilde{H}_g(-x,0,0))$ respectively. The isotriplet combination $(\tilde{E}_u-\tilde{E}_d)$ contains the pion pole and the flavor-singlet combination $(\tilde{E}_u+\tilde{E}_d+\tilde{E}_s)$ is sensitive to the axial U(1) problem [23,29] through the η' pole in the pseudoscalar form factor.

We now focus on the spin-dependent nonforward quark distribution $\tilde{F}_q$ to discuss the construction and renormalization of these distributions. The parton model and the light-cone correlation functions are normally formulated in the light-cone gauge $A_+=0$. Here the path-ordered exponential becomes a trivial unity factor and is dropped from the formalism:

$$\langle P'|\bar{q}(0)\gamma_+\gamma_5\mathcal{P}e^{i\int_{y_-}^0 dy_-A_+}q(y)|P\rangle \mapsto \langle P'|\bar{q}(0)\gamma_+\gamma_5 q(y)|P\rangle. \qquad (11)$$

In semiclassical QCD, before we come to discuss renormalization and anomalies, the gluonic degrees of freedom have dropped out along with the path-ordered exponential—hence the terminology "quark distribution."[1] Before we consider subtleties associated with anomaly theory—see Sec. IV below—the nonforward parton distributions have the following simple interpretation [6,4]. Expanding out the quark and gluon field operators in $q(y)$ and $G_{\mu\nu}(y)$ one finds the following. For $\zeta<x<1$ we have the situation where one removes a quark carrying light-cone momentum $k_+=xP_+$ and transverse momentum $\vec{k}_\perp$ from the initial state proton and re-inserts it into the final state proton with the same chirality but with light-cone momentum fraction $x-\zeta$ and transverse momentum $\vec{k}_\perp-\vec{\Delta}_\perp$. For $\zeta-1<x<0$ one finds the situation for removing an antiquark with momentum fraction $\zeta-x$ and re-inserting it with momentum fraction $-x$. For $0<x<\zeta$ the photons scatter off a virtual quark-antiquark pair in the initial proton wave function: the quark of the pair has light-cone momentum fraction x and transverse momentum $\vec{k}_\perp$, whereas the antiquark has light-cone momentum fraction $\zeta-x$ and transverse momentum $\vec{\Delta}_\perp-\vec{k}_\perp$. The third region $0<x<\zeta$ is not present in deep inelastic scattering where $\zeta=0$. The points $x=0$ and $x=\zeta$ correspond to zero-momentum modes. The flavor-singlet spin-dependent parton distributions at these two points are sensitive to the role of zero modes in dynamical $U_A(1)$ symmetry breaking [23,29], which have the potential to generate new nonperturbative contributions to the spin-dependent part of $\mathcal{A}_{\mathrm{DVCS}}$ beyond those appearing in the factorization formula (5). The $J=0$ Regge fixed pole [30] in the spin-independent part of $\mathcal{A}_{\mathrm{DVCS}}$ which generates a constant real term in $\mathcal{A}_{\mathrm{DVCS}}$ is manifest in Eqs. (5),(6) through the $\zeta\to 0$ limit of C_q.

The x moments, $\int_{\zeta-1}^{+1}dx\,x^n$, of the nonforward parton distributions are evaluated as follows. First one writes x^n as a derivative (in y_-) acting on $e^{ixP_+y_-/2}$. Integrating by parts (with respect to x) over the exponential yields a Dirac delta function $\delta(y_-)$. The y_- integral then projects out the nonforward matrix elements of local operators. One finds

[1] In QCD the "quark" and "gluon" distributions are scale dependent and mix under renormalization group or QCD evolution; only (partially) conserved, scale-invariant quantities which are not renormalized like the vector current $[\bar{q}\gamma_+q](0)$ and isotriplet axial-vector current $[\bar{u}\gamma_+\gamma_5u-\bar{d}\gamma_+\gamma_5d](0)$ can be regarded as "purely quark" or fermionic in the sense that they are insensitive to gluonic degrees of freedom under renormalization.

$$\int_{\zeta-1}^{1} dx\, x^n \tilde{F}_q(x,\zeta,t)$$
$$=\frac{1}{2}\left(\frac{2}{P_+}\right)^n \langle P'|\overline{q}(0)\gamma_+\gamma_5(i\partial_+)^n q(0)|P\rangle|_{y_+=y_\perp=0}$$
$$=\frac{1}{2\overline{P}_+}\overline{U}(P')\left[\int_{\zeta-1}^{+1} dx\, x^n \tilde{H}_q(x,\zeta,t)\gamma_+\gamma_5 + \int_{\zeta-1}^{+1} dx\, x^n \tilde{E}_q(x,\zeta,t)\frac{1}{2M}\gamma_5(-\Delta_+)\right]U(P). \quad (12)$$

A caveat is due here: renormalization means that we have to be careful not to simply set $y_-=0$ in the point-split operator to obtain the local operator. In QCD the bare light-cone correlation functions are ultra-violet divergent requiring renormalization [31]. This renormalization means that spin dependent (nonforward) parton distribution $\tilde{F}_q$ is sensitive to the axial anomaly.

We now discuss the effect of the anomaly in nonforward parton distributions.

IV. RENORMALIZATION AND ANOMALIES

Going beyond the semiclassical approximation to QCD we have to take into account that the renormalized composite operator

$$[\overline{q}\gamma_\mu\gamma_5 q]^R_{\mu^2}(0)\neq\overline{q}(0)\gamma_\mu\gamma_5 \quad \text{multiplied by } q(0). \quad (13)$$

This point is especially important when we evaluate the integral over y_- of the point-split matrix element with $\delta(y_-)$. Evaluating the moments of $\tilde{F}_q$ is nontrivial because the point-split operator is highly singular in the limit that the point splitting is taken to zero. In full QCD it is necessary to work with renormalized composite operators instead of their semi-classical prototypes. To see this explicitly consider Schwinger's derivation [32,33] of the axial anomaly using point split regularization. For $(z'-z'')\to 0$, one finds that the vacuum to vacuum matrix element of the point-split operator in an external gluon field is

$$\langle \text{vac}|[\overline{q}(z')\gamma_+\gamma_5 q(z'')]|\text{vac}\rangle \simeq \frac{ig}{8\pi^2}\tilde{G}_{+\nu}\frac{(z'-z'')^\nu}{(z'-z'')^2} \quad (14)$$

where the gluonic term arises from pinching a gluonic insertion between z' and z''. Going to the light cone ($z_T=0, z_+\to 0$) the factor

$$\frac{(z'-z'')_+}{(z'-z'')^2}\mapsto\frac{1}{(z'-z'')_-} \quad (15)$$

which diverges when $z'\to z''$. One clearly has to be careful and ensure that the theory and its interpretation does not assume equality of both sides in Eq. (13).

The problem is resolved [31] if, working in light-cone gauge, we define

$$\langle P',S'|[\overline{q}(0)\gamma_+\gamma_5 q(y_-)]^R_{\mu^2}|P,S\rangle_c \equiv \sum_n \frac{(iy_-)^n}{n!}\langle P',S'|[\overline{q}\gamma_+\gamma_5(iD_+)^n q]^R_{\mu^2}(0)|P,S\rangle_c \quad (16)$$

and

$$\langle P',S'|\mathrm{Tr}[G_{+\nu}(0)\tilde{G}_+^{\ \nu}(y_-)]^R_{\mu^2}|P,S\rangle_c \equiv \sum_n \frac{(iy_-)^n}{n!}\langle P',S'|\mathrm{Tr}[G_{+\nu}(iD_+)^n\tilde{G}_+^{\ \nu}]^R_{\mu^2}(0)|P,S\rangle_c . \quad (17)$$

That is, we treat the nonlocal operators in Eqs. (7)–(10) as a series expansion in terms of renormalized composite local operators in the operator product expansion. The superscript R denotes the renormalization prescription and μ^2 denotes the renormalization scale. The composite operators $[\overline{q}\gamma_+\gamma_5(iD_+)^{2n}q](0)$ and $\mathrm{Tr}[G_{+\nu}(iD_+)^{2n}\tilde{G}_+^{\ \nu}](0)$ mix under renormalization. The axial vector and higher-spin axial tensor operators are sensitive to the axial anomaly.

A. The axial anomaly

The gauge-invariantly renormalized flavor singlet axial-vector current

$$J^{GI}_{\mu 5}=[\overline{u}\gamma_\mu\gamma_5 u+\overline{d}\gamma_\mu\gamma_5 d+\overline{s}\gamma_\mu\gamma_5 s]^{GI}_{\mu^2} \quad (18)$$

satisfies the anomalous divergence equation [7,8]

$$\partial^\mu J^{GI}_{\mu 5}=2f\partial^\mu K_\mu+\sum_{i=1}^{f} 2im_i\overline{q}_i\gamma_5 q_i . \quad (19)$$

Here

$$K_\mu=\frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A^\nu_a\left(G^{\rho\sigma}_a-\frac{1}{3}gf_{abc}A^\rho_b A^\sigma_c\right)\right] \quad (20)$$

is a renormalized version of the gluonic Chern-Simons current and the number of light flavors f is 3. Equation (19) allows us to write

$$J^{GI}_{\mu 5}=J^{\text{con}}_{\mu 5}+2fK_\mu \quad (21)$$

where $J^{\text{con}}_{\mu 5}$ and K_μ satisfy the divergence equations

$$\partial^\mu J^{\text{con}}_{\mu 5}=\sum_{i=1}^{f} 2im_i\overline{q}_i\gamma_5 q_i \quad (22)$$

and

$$\partial^\mu K_\mu=\frac{g^2}{32\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}. \quad (23)$$

Here $(g^2/32\pi^2)G_{\mu\nu}\tilde{G}^{\mu\nu}$ is the topological charge density. The partially conserved current is scale invariant and the scale dependence of $J^{GI}_{\mu 5}$ is carried entirely by K_μ. When we make a gauge transformation U the gluon field transforms as

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \tag{24}$$

and the operator K_μ transforms as

$$K_\mu \to K_\mu + i\frac{g}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu(U^\dagger \partial^\alpha U A^\beta) + \frac{1}{24\pi^2}\epsilon_{\mu\nu\alpha\beta}[(U^\dagger\partial^\nu U)(U^\dagger\partial^\alpha U)(U^\dagger\partial^\beta U)]. \tag{25}$$

Gauge transformations shuffle a scale invariant operator quantity between the two operators $J^{\rm con}_{\mu 5}$ and K_μ while keeping $J^{GI}_{\mu 5}$ invariant.

The non-Abelian three-gluon part of K_+ vanishes in $A_+ = 0$ gauge and the forward matrix elements of K_+ are invariant under residual gauge degrees of freedom in this gauge. Furthermore the forward matrix elements of K_+ measure the amount of spin carried by gluonic partons in the target [12]. This leads ultimately to the "partons," AB and JET scheme decompositions of $g_A^{(0)}$ in Sec. II.[2] As soon as we move away from the forward direction matrix elements of K_+ will pick up a gauge-dependent contribution, which must decouple from any physical observable.

B. The axial anomaly and higher moments

The anomaly is also present in the $C=+1$ higher spin axial tensors [36]. In general, for a given choice of renormalization prescription R, the renormalized axial tensor operator differs from the gauge invariant operator by a multiple of a gauge-dependent, gluonic counterterm $K_{\mu\mu_1\ldots\mu_{2n}}$, viz.

$$\begin{aligned}[\bar{q}\gamma_\mu\gamma_5 iD_{\mu_1}\ldots iD_{\mu_{2n}}q]^R_{Q^2}(0) &= [\bar{q}\gamma_\mu\gamma_5 iD_{\mu_1}\ldots iD_{\mu_{2n}}q]^{GI}_{Q^2}(0) \\ &+ \lambda^{(K)}_{R,n}[K_{\mu\mu_1\ldots\mu_{2n}}]_{Q^2}(0) \\ &+ \lambda^{(G)}_{R,n}[G_{\mu\alpha}iD_{\mu_1}\ldots\tilde{G}^\alpha_{\mu_{2n}}]^{(GI)}_{Q^2}(0).\end{aligned} \tag{26}$$

Shifting between possible renormalization schemes, we pick up contributions from counterterms involving the gauge-invariant gluonic operators $G_{\mu\alpha}iD_{\mu_1}\ldots iD_{\mu_{2n-1}}\tilde{G}^\alpha_{\mu_{2n}}$ and also the gauge-dependent $K_{\mu\mu_1\ldots\mu_{2n}}$. The coefficients $\lambda^{(K)}_{R,n}$ and $\lambda^{(G)}_{R,n}$ are fixed by the choice of renormalization prescription. In $A_+=0$ gauge the two-gluon part of $K_{\mu\mu_1\ldots\mu_{2n}}$ reads

$$K_{+\ldots+(2n+1)} = \frac{\alpha_s}{\pi}\lambda^{(K)}_n \epsilon_{+\lambda\alpha\beta}A^\alpha \partial^\lambda (i\partial_+)^{2n}A^\beta. \tag{27}$$

It is not easy to derive the $K_{\mu\mu_1\ldots\mu_{2n}}$ beyond the two gluon term. Unlike the topological charge density $(\alpha_s/4\pi)G_{\mu\nu}\tilde{G}^{\mu\nu}$ the higher spin operators $(\alpha_s/4\pi)G_{\mu\nu}iD_{\mu_1}\ldots iD_{\mu_{2n}}\tilde{G}^{\mu\nu}$ are not topological invariants for $n\geq 1$. It follows that they are not total derivatives and, therefore, one cannot use a divergence equation alone to fix the non-Abelian part of $K_{\mu\mu_1\ldots\mu_{2n}}$. There is no equation $\partial^{\mu_j}\mathcal{SK}_{\mu\mu_1\ldots\mu_{2n}} = (\alpha_s/4\pi)G_{\mu\nu}iD_{\mu_1}\ldots iD_{\mu_{2n}}\tilde{G}^{\mu\nu}$ for $n\geq 1$.

V. THE AXIAL ANOMALY IN $\tilde{F}_Q$

The first observation to make is that the nonforward matrix elements of K_+ (and $K_{+\ldots+(2n+1)}$) are gauge dependent in $A_+=0$ gauge, even in perturbation theory. This means that these operators must decouple from any factorization scheme or "generalized operator product expansion" [3] for hard exclusive reactions like deeply virtual Compton scattering. This is in contrast to the situation in deep inelastic scattering where the forward matrix elements of K_+ are invariant under residual gauge degrees of freedom in $A_+=0$ gauge.

Since there is no gauge-invariant local gluonic operator with quantum numbers $J^{PC}=1^{++}$ it follows that the first moment of the spin-dependent *nonforward* gluonic coefficient must vanish. This holds true in the nonforward generalization of the $\overline{\rm MS}$ factorization scheme for handling the infrared mass singularities.[3] However, there is no gauge-invariant nonforward generalization of the popular JET and AB schemes used to describe polarized deep inelastic data. In these schemes there is an explicit gluonic contribution to the first moment of g_1 associated with the invariant contribution of K_+ (in $A_+=0$ gauge) to the forward matrix element of J^{GI}_{+5} induced by a nonvanishing first moment of the spin-dependent gluonic coefficient.

The gauge dependence of the nonforward matrix elements of K_+ also means that one has to be careful with the interpretation of the spin-dependent nonforward parton distributions in terms of amplitudes to extract and then re-insert a (well defined) parton with a given momentum. Consider the nonforward spin-dependent quark distribution defined first using gauge invariant renormalization and second via the partially conserved axial vector current. In the first case, glu-

[2] The current $J^{\rm con}_{\mu 5}$ and Δq in the "partons," AB and JET schemes are each renormalization scale invariant. One can define alternative scale invariant quantities $\Delta q_{\rm inv}$ through a linear-combination of the isotriplet and flavor-octet axial charges with the scale-invariant flavor-singlet axial-charge $g_A^{(0)}|_{\rm inv} = g_A^{(0)}|_{(\mu^2=\infty)}$ [34] which is defined by factoring out the multiplicative renormalization group factor [23,35], denoted $E(\alpha_s)$ in [13], from $J^{GI}_{\mu 5}$. These $\Delta q_{\rm inv}$ still contain gluonic information through the contributions of $[-(\alpha_s/2\pi)\Delta g]_{(\mu^2=\infty)}$ and $\mathcal{C}$ in Eq. (2).

[3] See, e.g., the calculations of Mankiewicz *et al.* [37] and Ji and Osborne [3]; in the notation of Mankiewicz *et al.* [37] $(\partial/\partial u)C_1^{A,g}|_{u=0}=0$.

onic information is intrinsically built into the definition of the "polarized quark" via the axial anomaly. In the second, the notion of "polarized quark parton" is no longer gauge invariant. Taken together, this means that one has to be careful how far one carries through the semi-classical interpretation of parton distributions. The gluonic information built into the flavor-singlet part of $\tilde{F}_q$ is manifest through the massive η' and absence of any (nearly) massless pseudo-Goldstone axial U(1) pole in $\tilde{E}_q$.

The zero modes associated with dynamical $U_A(1)$ symmetry breaking mean that the perturbative QCD formula (5) for deeply virtual Compton scattering may not necessarily exhaust the total cross section. For example, in Crewther's theory of quark-instanton interactions one would expect $\delta(x)$ and $\delta(x-\zeta)$ contributions in the flavor-singlet part of $\tilde{H}_q(x,\zeta,t)$ associated with the transfer of incident axial charge carried by quarks and antiquarks respectively from partons with finite momentum to zero modes. Such zero mode contributions are not easily reconcilable with the perturbative QCD factorization expression (5), in particular the singularities in the leading-twist coefficient function $\tilde{C}_q$ in Eq. (6). This problem is not relevant to isovector pion or rho production [38] which is described just in terms of the isovector nonforward distributions. Any flavor-singlet zero modes will not contribute to these processes.

VI. ORBITAL ANGULAR MOMENTUM

A sum rule [1,40] relates the form factors appearing in F_q and F_g in the spin-independent part of $\mathcal{A}_{\rm DVCS}$ to the quark and gluonic total angular-momentum contributions to the spin of the proton. The second moments of F_q and F_g project out the nonforward matrix elements of the QCD energy momentum tensor:

$$\begin{aligned}\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\bar{P}^{(\mu}i\sigma^{\nu)\alpha} \\ \times \Delta_\alpha/2M + C_{q,g}(t)\Delta^{(\mu}\Delta^{\nu)}/M + \mathcal{O}(\Delta^3)] \\ \times U(P). \end{aligned} \tag{28}$$

There are no massless bosons which couple to $T^{\mu\nu}$ in QCD, which is associated with the fact that Poincaré invariance is not spontaneously broken in QCD (with corresponding Goldstone bosons). This means that the expansion in Δ and the forward limit of the form factors in Eq. (28) are well defined: there are no $\Delta_\mu\Delta_\nu/\Delta^2$ terms. The current associated with Lorentz transformations is

$$M_{\mu\nu\lambda} = z_\nu T_{\mu\lambda} - z_\lambda T_{\mu\nu}. \tag{29}$$

The total angular momentum operator is related to the energy-momentum tensor by

$$J^z_{q,g} = \left\langle P', \frac{1}{2} \right| \int d^3z (\vec{z}\cdot\vec{T}_{q,g})^z \left| P, \frac{1}{2} \right\rangle. \tag{30}$$

Substituting Eq. (28) into Eq. (30) and taking the forward limit $\Delta\to 0$ one obtains

$$J^z_{(q,g)} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]. \tag{31}$$

This result is Ji's sum rule [1] relating total angular momentum to the form factors measured appearing in Eqs. (7) and (8).

If one can extract $J_{q,g}$ from the forward limit of $A(t)$ and $B(t)$ in hard exclusive processes, then subtracting S_z from polarized deep inelastic scattering would give information about the orbital angular momentum L_z. Whereas the intrinsic spin S_z^q is sensitive to the axial anomaly, the total angular momentum $J_z^{(q+g)}$ is not because it is measured by a conserved current. Theoretical studies [19,39] show that J_z^q and J_z^g are each anomaly free in perturbative QCD meaning that the axial anomaly cancels between S_z^q and L_z^q in perturbation theory [19,39,40].

This result generalizes beyond perturbation theory [40] with the added consequence that there is no zero-mode contribution to J_q and J_g. To see this, first consider the crossing symmetry in x of the spin-independent quark distributions. The second moment of the charge parity plus distribution $(q+\bar{q})(x)$ projects out the nucleon matrix element of the energy-momentum tensor $T_{\mu\nu}$. Any zero mode contribution to the right-hand side of the energy-momentum sum rule would be associated with a $\delta'(x)$ term in $(q+\bar{q})(x)$. Less singular $\delta(x)$ terms may be induced in the spin-dependent distributions by quark instanton interactions. I know of no model which predicts a stronger singularity like $\delta'(x)$. Phenomenologically, such a term in the spin-independent structure function F_2 would lead to a violation of the energy-momentum sum rule for partons, which is not observed. Since there is no zero-mode contribution to J_z it follows that any zero-mode contribution to $g_A^{(0)}$ is compensated by a second zero mode with equal magnitude but opposite sign in the orbital angular momentum L_z^q. This has the practical consequence that one has to be careful how one interprets any determination of L_z^q from DVCS through the sum rule (31). The orbital angular momentum carried by constituent quarks and by "current quark" partons are not necessarily the same, and is distinguished by the topological term $\mathcal{C}$ measurable in νp elastic scattering. What happens to spin and orbital angular momentum in the transition from current to constituent quarks is intimately related to the dynamics of axial U(1) symmetry breaking.

Finally, we note that in perturbative QCD one also has to be careful to quote any value of "L_z^q" with respect to the factorization scheme used to extract "S_z^q" from polarized deep inelastic data. For example, $\Delta q_{\overline{\rm MS}} = (\Delta q - (\alpha_s/2\pi)\Delta g)_{\rm AB,JET}$—see also Shore [41].

VII. CONCLUSIONS

In summary, the quark total angular momentum J_q measurable through deeply virtual Compton scattering is independent of the QCD axial anomaly. This J_q is defined through the proton matrix element of the QCD angular-momentum tensor. Gluonic QCD axial anomaly effects occur with equal magnitude and opposite sign in the orbital L_q and intrinsic S_q contributions to J_q. This means that going from J_q (anomaly free) to the orbital angular momentum L_q one has to be careful to quote the perturbative QCD factorization scheme and process (polarized deep inelastic or νp elastic scattering) used to extract information about the intrinsic spin contribution S_q. The axial anomaly also induces gluonic information in the spin-dependent, flavor-singlet nonforward parton distribution $(\tilde{F}_u+\tilde{F}_d+\tilde{F}_s)$, which appears in the spin-dependent part of the scattering amplitude $\mathcal{A}_{\mathrm{DVCS}}$. This gluonic information is manifest in the large-mass η' pole in $(\tilde{E}_u+\tilde{E}_d+\tilde{E}_s)$ associated with the flavor-singlet pseudoscalar form factor.

ACKNOWLEDGMENTS

It is a pleasure to thank S.J. Brodsky, A. Donnachie, P.V. Landshoff, G. Piller, A. Radyushkin and W. Weise for helpful discussions. This work was supported in part by DFG.

[1] X. Ji, Phys. Rev. Lett. **78**, 610 (1997).
[2] A.V. Radyushkin, Phys. Rev. D **56**, 5524 (1997).
[3] X. Ji, Phys. Rev. D **55**, 7114 (1997); X. Ji and J. Osborne, *ibid.* **57**, R1337 (1998); **58**, 094018 (1998).
[4] S.J. Brodsky, M. Diehl, and D.S. Hwang, Nucl. Phys. **B596**, 99 (2001).
[5] A.V. Radyushkin, hep-ph/0101225; Acta Phys. Pol. B **30**, 3647 (1999).
[6] X. Ji, J. Phys. G **24**, 1181 (1998).
[7] S.L. Adler, Phys. Rev. **177**, 2426 (1969).
[8] J.S. Bell and R. Jackiw, Nuovo Cimento A **60**, 47 (1969).
[9] R. Windmolders, Nucl. Phys. B (Proc. Suppl.) **79**, 51 (1999).
[10] G.M. Shore and Zuoz lecture, hep-ph/9812354; Erice lecture, hep-ph/9812355.
[11] R.L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).
[12] R.L. Jaffe, Phys. Lett. B **365**, 359 (1996).
[13] S.D. Bass, Eur. Phys. J. A **5**, 17 (1999).
[14] H. Fritzsch, Phys. Lett. B **229**, 122 (1989); **256**, 75 (1991).
[15] S. Forte, Nucl. Phys. **B331**, 1 (1990).
[16] R.L. Jaffe, hep-ph/0102281.
[17] G. Altarelli and G.G. Ross, Phys. Lett. B **212**, 391 (1988).
[18] A.V. Efremov and O.V. Teryaev, JINR Report E2–88–287 (1988).
[19] R.D. Carlitz, J.C. Collins, and A.H. Mueller, Phys. Lett. B **214**, 229 (1988).
[20] S.D. Bass, B.L. Ioffe, N.N. Nikolaev, and A.W. Thomas, J. Mosc. Phys. Soc. **1**, 317 (1991).
[21] S.D. Bass, Mod. Phys. Lett. A **13**, 791 (1998).
[22] S.D. Bass and A. De Roeck, Nucl. Phys. B (Proc. Suppl.) **105**, 1 (2002); B. Lampe and E. Reya, Phys. Rep. **332**, 1 (2000).
[23] R.J. Crewther, Acta Phys. Austriaca, Suppl. **19**, 47 (1978).
[24] R.D. Ball, S. Forte, and G. Ridolfi, Phys. Lett. B **378**, 255 (1996).
[25] E. Leader, A.V. Sidorov, and D.B. Stamenov, Phys. Lett. B **445**, 232 (1998).
[26] G.T. Bodwin and J. Qiu, Phys. Rev. D **41**, 2755 (1990).
[27] H.Y. Cheng, Phys. Lett. B **427**, 371 (1998); D. Muller and O.V. Teryaev, Phys. Rev. D **56**, 2607 (1997).
[28] A.V. Radyushkin, Phys. Lett. B **449**, 81 (1999).
[29] G. 't Hooft, Phys. Rep. **142**, 357 (1986).
[30] S.J. Brodsky, F.E. Close, and J.F. Gunion, Phys. Rev. D **6**, 1384 (1972); **8**, 3578 (1973); P.V. Landshoff and J.C. Polkinghorne, *ibid.* **5**, 2056 (1972); D.J. Broadhurst, J.F. Gunion, and R.L. Jaffe, Ann. Phys. (N.Y.) **81**, 88 (1973).
[31] C.H. Llewellyn Smith, in *Symmetry Violations in Subatomic Physics*, Proceedings of the CAP-NSERC Summer Institute, edited by B. Castel and P.J. O'Donnell (World Scientific, Singapore, 1989), p. 139; C.H. Llewellyn Smith, hep-ph/9812301.
[32] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[33] R. Jackiw, in *Current Algebra and Anomalies*, edited by S.B. Treiman *et al.* (World Scientific, Singapore, 1985), pp. 130–136.
[34] S.D. Bass, R.J. Crewther, F.M. Steffens, and A.W. Thomas, hep-ph/9701213.
[35] J. Kodaira, Nucl. Phys. **B165**, 129 (1980).
[36] S.D. Bass, Z. Phys. C **55**, 653 (1992).
[37] L. Mankiewicz *et al.*, Phys. Lett. B **425**, 186 (1998); hep-ph/9712251 (revised).
[38] L. Mankiewicz, G. Piller, and A. Radyushkin, Eur. Phys. J. C **10**, 307 (1999); L. Mankiewicz, G. Piller, and T. Weigl, *ibid.* **5**, 119 (1998).
[39] X. Ji, J. Tang, and P. Hoodbhoy, Phys. Rev. Lett. **76**, 740 (1996).
[40] G.M. Shore and B.E. White, Nucl. Phys. **B581**, 409 (2000).
[41] G.M. Shore, Nucl. Phys. B (Proc. Suppl.) **96**, 171 (2001).